The Hubble Diagram of Type Ia Supernovae in Non-Uniform Pressure Universes

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ABSTRACT

We use the redshift-magnitude relation, as derived by Dąbrowski (1995), for the two exact non-uniform pressure spherically symmetric Stephani universes with the observer positioned at the center of symmetry, to test the agreement of these models with recent observations of high redshift type Ia supernovae (SNIa), as reported in Perlmutter et al. (1997). By a particular choice of model parameters, we show that these models can give an excellent fit to the observed redshifts and (corrected) B band apparent magnitudes of the Perlmutter et al. data, but for an age of the Universe which is typically about two Gyr – and may be more than three Gyr – greater than in the corresponding Friedmann model, for which non-negative values of the deceleration parameter appear to be favoured by the data. We show that this age increase is obtained for a wide range of the non-uniform pressure parameters of the Stephani models. We claim this paper is the first attempt to compare inhomogeneous models of the universe with real astronomical data.

Several recent calibrations of the Hubble parameter, from the Hubble diagram of SNIa and other distance indicators, indicate a value of $H_0 \simeq 65$, and a Hubble time of ~ 15 Gyr. Based on this value for H_0 and assuming $\Lambda \geq 0$, the P97 data would imply a Friedmann age of at most 13 Gyr and in fact a best-fit (for $q_0 = 0.5$) age of only 10 Gyr. Our Stephani models, on the other hand, can give a good fit to the P97 data with an age of up to 15 Gyr. The Stephani models considered here could, therefore, significantly alleviate the conflict between recent cosmological and astrophysical age predictions. The choice of model parameters is quite robust: in order to obtain a good fit to the current P97 data,

one requires only that the non-uniform pressure parameter, a, in one of the models is negative and satisfies $|a| \lesssim\ 3$ km^2 s^{-2} Mpc^{-1}. This limit gives a value for the acceleration scalar, $\dot{u}$, of order $|\dot{u}| \lesssim 0.66 \times 10^{-10}\, r$ Mpc^{-1}, where r is the radial coordinate in the model. Thus, although the pressure is not zero at the center of symmetry, $r = 0$, the effect of acceleration is non-detectable at the center since the acceleration scalar vanishes there. However, the effect of the non-uniform pressure on the redshift-magnitude relation is clearly seen since neighbouring galaxies are not situated at the center and they necessarily experience acceleration. By allowing slightly larger, negative, values of a one may 'fine tune' the model to give an even better fit to the P97 data.

Subject headings: Cosmology - age of the Universe - supernovae - relativity

1. Introduction

The standard isotropic Friedmann cosmological models have naturally been the most widely investigated models in studies of the large-scale structure of the Universe. This is hardly surprising, in view of their mathematical simplicity and their generic prediction of an approximately linear Hubble expansion at low redshift, which is in excellent agreement with observational data (c.f. Strauss & Willick 1995; Postman 1997). Even in Friedmann models, however, the relation between apparent magnitude and log redshift is in general non-linear at higher redshift and depends explicitly on the spatial curvature of the Universe – or equivalently on the deceleration parameter, q_0.

For several decades astronomers have attempted to use the Hubble diagram of some suitable 'standard candle' (e.g. first-ranked cluster galaxies) to place constraints on the global geometry of the universe by comparing the observed redshift-magnitude relation of the standard candle with that predicted in Friedmann models with different values of q_0 (c.f. Peach 1970; Gunn & Oke 1975; Schneider, Gunn & Hoessel 1983; Sandage 1988). Results from such analyses have thus far proved inconclusive, however. Due to the intrinsic dispersion in the luminosity function of the standard candles available, one previously had to reach at least $z \simeq 1$ before the predictions of models with different values of q_0 became sufficiently distinct to be detectable; at the same time, however, the effects of luminosity and number density evolution also become important at these redshifts, and are very difficult to correct for. The situation for the Hubble diagram of quasars is equally – if not even more – problematical. Tonry (1993) suggested that the contraints on q_0 from such studies were no better than $-1 < q_0 < 1$.

Recently, however, it has been suggested that type Ia supernovae (hereafter

SNIa) represent a standard candle of sufficiently small dispersion to allow meaningful estimates of q_0 now to be derived from the SNIa Hubble diagram at more moderate redshift. In Perlmutter et al. (1997; hereafter P97) a preliminary analysis is presented of seven distant SNIa in the range $0.35 < z < 0.50$. A comparison of the SNIa magnitudes and redshifts with the predicted relation for various Friedmann models appears to exclude large negative values of q_0, and is best fitted by values close to $q_0 = 0.5$. This poses a potentially serious problem for Friedmann models. Since many recent determinations of the Hubble constant (including a number of analyses using SNIa) suggest that H_0 lies in the range 65 – 70, this would imply an age of the universe of less than 10 Gyr in the 'standard' $\Omega_0 = 1, \Lambda = 0$ scenario. This result would appear to be in sharp conflict with recent astrophysical age determinations from e.g. globular clusters and white dwarf cooling (c.f. Chaboyer 1995; Hendry & Tayler 1996) – a conflict which is only slightly alleviated by revisions to globular cluster age estimates in the light of results from the HIPPARCOS satellite (Chaboyer et al. 1997). Reducing the value of Ω_0 lessens the conflict somewhat, but agreement is still only marginal if one accepts a robust lower bound for the matter density of $\Omega_{\rm m} = 0.3$, as has been suggested by several different methods of analysing large-scale galaxy redshift surveys (c.f. Strauss & Willick 1995). This situation has helped to give a renewed impetus to models with a positive cosmological constant (c.f. Liddle et al. 1996) which contributes an additional component, Ω_Λ, to make up the critical density and at the same time extends the age of the Universe by up to 2 Gyr – depending on the value of H_0 and $\Omega_{\rm m}$. However, because of the relation

$$q_0 = \frac{1}{2}\Omega_{\rm m} - \Omega_\Lambda, \tag{1}$$

it is clear that a positive value of q_0 is incompatible with a positive value of Λ unless the matter density is at least two-thirds of the critical density. The $q_0 = 0$ case, assuming $\Omega_{\rm m} = 2/3$, $\Omega_\Lambda = 1/3$ and $H_0 = 65$, would give an age of the Universe of

just over 11 Gyr; as q_0 increases the age is decreased still further. Thus, if the results of P97 prove to be correct and the deceleration parameter *is* non-negative, then the conflict between cosmological and astrophysical age predictions remains firmly unresolved – at least if $H_0 \gtrsim 65$. Independent results showing that a positive value of Λ is incompatible with the so-called VLBI data (Kellermann 1993), using the angular diameter test, were obtained by Krauss & Schramm (1993) and Stelmach (1994).

In this paper we propose one method to alleviate this age conflict by considering some inhomogeneous cosmological models in which the relation between the age of the Universe and a generalised Hubble constant is more general than in the Friedmann case. Despite some theoretical plots of the observational quantities for inhomogeneous models (e.g. Goicoechea & Martin-Mirones 1987, Moffat & Tatarski 1995, Dąbrowski 1995, Humphreys, Maartens & Matravers 1997) this paper is – as far as we are aware – the first to compare these models with real astronomical data. In particular, we show that taking an inhomogeneous model into account allows us to obtain a good fit between the predicted redshift-magnitude relation and P97 data, but for an age of the Universe which is several Gyr older than in the Friedmann case. The models under consideration have been discussed before and are known as Stephani Universes (c.f. Kramer et al. 1980; Krasiński 1983; Dąbrowski 1993). In these models the energy density ϱ depends just on the cosmic time, similarly to the Friedmann models, but the pressure, p, is a function of both spatial coordinates and a time coordinate; hence the models are usually referred to as 'inhomogeneous pressure universes'. In the spherically symmetric case under consideration, the pressure is just a function of time and radial coordinate which means that its values are the same on spheres, $r =$ constant, around the center of symmmetry but differ from sphere to sphere. This essentially means that there is a spatial pressure

gradient and particles are accelerated in the direction from high-pressure regions to low-pressure regions. This effect is usually described by the acceleration vector $\dot{u}_r$ which in the case of spherical symmetry has only one (radial) component, or by the acceleration scalar $\dot{u}$ (cf. Eqs. 2.18 and 2.23 of Dąbrowski 1995). The acceleration represents the combined effect of gravitational and inertial forces on the fluid which, in fact, similarly as in Newtonian physics, are unable to be separated. As a first approximation we assume the observer is placed at the center of symmetry, which results in no pressure gradient at the observer's postion. This, in a sense, contradicts the Copernican Principle, but can easily be overcome by applying the formulae for a non-centrally observer given in Section 5 of Dąbrowski (1995) – an appropriate generalisation once larger samples of high-redshift supernovae become available.

In the standard approach we neglect the effect of pressure (i.e. we take pressureless dust – with pressure $p = 0$) as we evaluate chaotic velocities of galaxies to be small. This results in taking the acceleration, $\dot{u}$, also to be zero (cf. Ellis, 1971, for a discussion of the relation between these quantities). However, if there was large flux of neutrinos or gravitational waves for instance, this assumption would not be correct and we would need to take radiation pressure ($p = \frac{1}{3}\varrho$) into account. This has been of course investigated for isotropic cosmologies (Dąbrowski & Stelmach 1986, 1987) and all the observational quantities have been found. Our main point here is, however, that early universe processes such as phase transitions (for details see Vilenkin 1985, Kolb & Turner 1990, Vilenkin & Shellard 1994) may result in having different exotic types of matter (e.g. cosmic strings) with many different types of equations of state. In the easiest case (straight cosmic strings) they may end up with the exotic equation of state $p = -\frac{1}{3}\varrho$, but in general, the equation of state can be more complicated (e.g. Vilenkin & Shellard 1994 and de Vega & Sanchez 1994, in the context of superstrings) or spatially dependent (e.g. Narlikar, Pecker & Vigier

1991). The latter would especially be the case of our interest. Of course the standard energy conditions of Hawking and Penrose might be violated (cf. Hawking & Ellis 1973) which also happens for inflationary models for instance and our considerations here are, in a sense, on the same footing as those phenomena. As for the Stephani models, which do not admit any global barotropic equation of state, it has been shown that there exists a consistent nonbarotropic equation of state and the full thermodynamical scheme exists (Quevedo & Sussman 1995, Krasiński, Quevedo & Sussman 1997).

Regardless of the physical background of the models under consideration, one of our main tasks here is to draw attention to the entire class of inhomogeneous models which could be a useful alternative to Friedmann models in helping to resolve the apparent incompatibility of measurements of the Friedmann cosmological parameters. Even if the final outcome (after a thorough comparison with data) shows that the universe is indeed isotropic and homogeneous this conclusion must be drawn by applying some 'averaging scale-dependent procedures' (cf. Ellis 1984, Buchert 1997) since we evidently cannot see the universe to be like that on smaller scales. Being spherically symmetric, Stephani models can also be applied to a local underdense/overdense spherical region embedded in a globally isotropic Friedmann universe (Moffat & Tatarski 1995) in some analogy to the so-called 'Swiss Cheese' model (Kantowski, Vaughan & Branch 1995).

The reader interested in more generic models should be referred to the recent review by Krasiński (1997), as well as to some earlier papers concerning the most popular generalization of the Friedmann models such as the spherically symmetric Tolman Universes which are inhomogeneous density pressure-free dust shells (Tolman 1934; Bondi 1947; Bonnor 1974). Their properties have been studied quite thoroughly in Hellaby & Lake (1984, 1985) and Hellaby (1987, 1988) and

the observational relations for Tolman models were studied by Goicoechea & Martin-Mirones (1987), Moffat & Tatarski (1995) and quite recently by Humphreys, Maartens & Matravers (1997). However, in none of these cases has a comparison with real astronomical data been carried out.

The outline of this paper is as follows. In Section 2 we reproduce the redshift-magnitude relations for the two Stephani models considered here, as recently derived in Dąbrowski (1995). In Section 3 we briefly describe the SNIa data of P97. In Section 4 we fit these data to the redshift-magnitude relations of both Friedmann and Stephani models and thus obtain best-fit values for the model parameters. We then discuss the results of these fits and compare the age of the Universe given by the best-fit model parameters in the Friedmann and Stephani cases. Finally in Section 5 we summarise our conclusions.

2. The Redshift-magnitude relation for inhomogeneous pressure models

Recently Dąbrowski (1995) has considered the redshift-magnitude relation for Stephani universes. Two exact cases were presented and the predicted relations were plotted for a range of different parameter values. The relations were defined following the method of Kristian & Sachs (1966), of expanding all relativistic quantities in power series and truncating at a suitable order. Approximate formulae, to first order in redshift z, for Model I and Model II respectively were given by

$$\begin{aligned} m_{\rm B} &= M_{\rm B} + 25 + 5\log_{10}\left[cz\left(\frac{a\tau_0^2 + b\tau_0 + d}{2a\tau_0 + b}\right)\right] \\ &+ 1.086\left[1 + 4a\frac{(a\tau_0^2 + b\tau_0 + d)}{(2a\tau_0 + b)^2}\right] z \end{aligned} \qquad (2)$$

and

$$m_{\rm B} = M_{\rm B} + 25 - 5\log_{10}\frac{2}{3\tau_0} + 5\log_{10} cz$$

$$+ \quad 1.086 \left(\frac{1}{2} + \frac{9}{8} c^2 \alpha \tau_0^{\frac{4}{3}} \right) z, \qquad (3)$$

which are essentially equations (5.6) and (5.10) respectively of Dąbrowski (1995). [1] Here $m_{\rm B}$ and $M_{\rm B}$ denote apparent and absolute magnitude respectively in the B band, τ_0 denotes the current age of the Universe and $c = 3 \times 10^5$ km s^{-1} is the velocity of light. The constants a, b and d are parameters of Model I and α is a parameter of Model II. Convenient units for these parameters are: $[a]$ = km^2 s^{-2} Mpc^{-1}, $[b]$ = km s^{-1}, $[d]$ = Mpc and $[\alpha c^2]$ = (km s^{-1} Mpc)$^{-\frac{4}{3}}$. From the definition of the acceleration scalar (c.f. Dąbrowski 1995) we conclude that the parameters which relate directly to the non-uniformity of the pressure are a in Model I and α in Model II. In Model I, b plays a similar role to the coefficient of time in the expression for the scale factor ($R \propto \tau^p$ where p is any power) in Friedmann models and can be considered as an immanently Friedmannian parameter of the Stephani models, while a and α are completely non-Friedmannian.

The models considered here are spherically symmetric, which means that we can have both centrally placed and non-centrally placed observers. For simplicity the redshift-magnitude relations reproduced above correspond to a centrally placed observer. Dąbrowski (1995) also derived relations for the case of a non-centrally placed observer which, although more general, introduced several additional free parameters. The main difference in this more general case is that the apparent magnitude depends on the position of the source in the sky, and renders comparison

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[1] But written now for B band observations, which are the case under study in this paper. Note also that we have corrected a typographical error which appeared in Dąbrowski (1995) – the factor of 4 in the final term of Eq. (2) replacing the factor of 2 in Eq. (5.6) of that paper. We thank Chris Clarkson for drawing our attention to this error.

with the Friedmann case more complicated. We thus consider only centrally placed observers in this paper.

Note that these formulae are truncated at first order in z and thus would become increasingly inaccurate if applied to redshifts greater than or equal to unity. Since the redshifts of the P97 preliminary data extend only to $z \sim 0.5$ we proceed with the first-order expressions, and will also use the first-order expression for Friedmann models when comparing the fits. In future work, as the database of high redshift SNIa grows, we will extend the approximate redshift-magnitude relations to higher order, as required.

The reader is referred to Dąbrowski (1995) for a detailed discussion of the derivation of the above formulae. Note, however, that for Model I the expression for the generalised scale factor, $R(\tau)$, as a function of cosmic time, τ, is given by (Eq. 2.11 of Dąbrowski 1995)

$$R(\tau) = a\tau^2 + b\tau + d. \tag{4}$$

However, $R(\tau)$ does not have to be positive (Dąbrowski 1993, Section IV.A and Fig. 6) for the Stephani models. For the subclass under consideration, $R(\tau)$ easily relates to the spatial curvature of the models

$$k(\tau) = -4\frac{a}{c^2}R(\tau), \tag{5}$$

and the curvature index is not constant in time as for Friedmann models. In principle, one can restrict $R(\tau)$ to be positive (this is especially reasonable, if we want to obtain the full Friedmann limit), which ends up with the simple relation for the spatial curvature of the models being positive for negative non-uniform pressure parameter a and negative for positive a. Since $R(\tau) = 0$ at the singularity (the Big-Bang) we can require that $R(\tau) \to 0$ as $\tau \to 0$ (i.e. we set the origin of our time coordinate at the Big Bang) and thus demand that d is identically zero. Therefore,

according to condition (2.13) of Dąbrowski (1995), which, in fact, allows one to have the Friedmann limit for the Stephani models under consideration, we have

$$b^2 = 1, \tag{6}$$

for the values of b. Without loss of generality we assume that $b = +1$ (cf. the discussion above about the meaning of b in Friedmann models) – leaving only one free parameter of the Model I, which is a.

Other important physical quantities of Model I are as follows: the energy density

$$\frac{8\pi G}{c^4}\varrho(\tau) = \frac{3}{(a\tau^2 + b\tau)^2}, \tag{7}$$

the pressure

$$\frac{8\pi G}{c^2}p(\tau) = -\frac{1}{(a\tau^2 + b\tau)^2}\left[1 + 2a\left(a\tau^2 + b\tau\right)r^2\right], \tag{8}$$

and the acceleration scalar

$$\dot{u} = -2\frac{a}{c^2}r. \tag{9}$$

From the above one can see that the finite density singularities of pressure appear at $r \to \infty$ [2] where there is the antipodal center of symmetry. We assume that we are placed at the center of symmetry at $r = 0$ so we have these singularities far away from us (see Fig. 6 of Dąbrowski 1993). Of course we cannot live at the singularity of pressure.

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[2]Note that in Stephani models we use the so-called isotropic radial coordinate which results in the Friedmann metric to be taken in the isotropic form. It relates to the commonly used nonisotropic coordinate r_N via the relation $r_N = r/(1 + (1/4)r^2)$ (for details see Section II of Dąbrowski 1993). Because of isotropy at every point one cannot differentiate the centers $r = 0$ and $r = \infty$ in a Friedmann universe, which is not the case for the Stephani models.

At the center of symmetry the fluid fulfils the barotropic equation of state $p = -\frac{1}{3}\varrho$ (as for straight cosmic strings, cf. Vilenkin 1985), while at $r \to \infty$ the pressure goes to either plus or minus infinity. Then, assuming $R(\tau) > 0$, it diverges to minus infinity if $a > 0$, and to plus infinity if $a < 0$. In such a case, the particles are accelerated away from a high pressure region at $r = 0$ to low pressure regions at $r \neq 0$, if $a > 0$, and toward a low pressure region at $r = 0$ from high pressure regions at $r \neq 0$, if $a < 0$. Of course if $R(\tau) < 0$ the situation is the opposite. The acceleration scalar is zero at $r = 0$ and it diverges at $r \to \infty$.

In the case of Model II the time-dependent curvature index is given by (β plays here the same 'Friedmannian' role as b in Model II)

$$k(\tau) = -\alpha\beta^2 c^{-2}\tau^{\frac{2}{3}}, \tag{10}$$

while the energy density, pressure and acceleration scalar are given respectively by (Dąbrowski 1993, Appendix C)

$$\frac{8\pi G}{c^4}\varrho(\tau) = \frac{4}{3}\frac{1}{\tau^2} - \frac{3\alpha}{\tau^{\frac{2}{3}}}, \tag{11}$$

$$\frac{8\pi G}{c^2}p(\tau) = \frac{2\alpha}{\tau^{\frac{2}{3}}} - \frac{4}{3}\frac{\alpha\beta^2}{\tau^{\frac{4}{3}}}r^2 + \alpha^2\beta^2 r^2, \tag{12}$$

and

$$\dot{u} = -\frac{1}{2}\alpha\beta r. \tag{13}$$

3. The SNIa observations of Perlmutter et al. (1997)

Type Ia supernovae are thought to be the result of the thermonuclear disruption of a white dwarf star which has accreted sufficient matter from a binary companion to reach the Chandrasekhar mass limit. For several decades they have been considered as suitable (nearly) standard candles for the testing of cosmological models because

of the relatively small dispersion of their luminosity function at maximum light and the fact that they are observable at very great distances. In recent years the Hubble diagram of SNIa has been used by a number of authors to obtain estimates of the Hubble constant (c.f. Riess, Press & Kirshner 1996; Hamuy et al. 1995, 1996; Branch et al. 1996) and the motion of the Local Group (Riess, Press & Kirshner 1995). P97 consider the redshift-magnitude relation of SNIa at high redshift, observed by the 'Supernova cosmology project', as a means of constraining q_0. In P97 SNIa are not treated as precise standard candles, but a 'stretch factor' correction is applied to account for the correlation between SNIa luminosity and the shape of their light curve.

In this paper we use the redshifts and B band magnitudes – with and without light curve shape corrections – as presented in Table 1 of P97, to which the reader is referred for details of their observing strategy, data reduction procedures and magnitude error estimates.

4. Comparison of the data with Friedmann and Stephani Models: results and discussion

4.1. Friedmann Models

Figure 4 of P97 shows the Hubble diagram of their SNIa compared with the theoretical magnitude-redshift relations for a Friedmann model with different combinations of $\Omega_{\rm m}$ and Ω_Λ. While P97 argue correctly that one should generally express the Friedmann magnitude-redshift relations in terms of $\Omega_{\rm m}$ and Ω_Λ separately, and not just in terms of their combination via q_0, for the redshift range

of the P97 data one may adequately approximate the relation by

$$m_{\mathrm{B}} = \mathcal{M}_{\mathrm{B}} + 5\log_{10} cz + 1.086(1 - q_0)z, \tag{14}$$

where

$$\mathcal{M}_{\mathrm{B}} = M_{\mathrm{B}} - 5\log_{10} H_0 + 25, \tag{15}$$

with the corresponding expression for the corrected B band magnitudes. For reasons which will become clear when we consider the Stephani models, it is useful for us to write Eq. (14) in this form, in terms of q_0. Since we will make use of similar expressions for the Stephani universes, we construct for the Friedmann case the (reduced) chi-squared statistic

$$\chi^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left[\frac{m_{\mathrm{B}}^{\mathrm{obs}}(i) - m_{\mathrm{B}}^{\mathrm{pred}}(i)}{\sigma(i)}\right]^2, \tag{16}$$

where n is the number of SNIa, $m_{\mathrm{B}}^{\mathrm{obs}}(i)$ and $\sigma(i)$ are respectively the observed B band apparent magnitude and error estimate of the ith SNIa, and $m_{\mathrm{B}}^{\mathrm{pred}}(i)$ is the predicted B band apparent magnitude of the ith SNIa, for a given value of q_0, derived from equation 5 (or its equivalent for the corrected magnitudes). Following P97 we adopt $\mathcal{M}_{\mathrm{B}} = -3.17 \pm 0.03$ and $\mathcal{M}_{\mathrm{B,corr}} = -3.32 \pm 0.05$.

From Eqs. (14) and (16) it follows that $\hat{q}_0$, the maximum likelihood (equivalently minimum χ^2) estimate of q_0, is given by:-

$$\hat{q}_0 = -\left[\sum_{i=1}^{n}\frac{x_i y_i}{\sigma^2(i)}\right]\left[\sum_{i=1}^{n}\frac{x_i^2}{\sigma^2(i)}\right]^{-1}, \tag{17}$$

where

$$x_i = 1.086z(i) \tag{18}$$

and

$$y_i = m_{\mathrm{B}}^{\mathrm{obs}}(i) - \mathcal{M}_{\mathrm{B}} - 5\log cz(i) - 1.086z(i). \tag{19}$$

Substituting the appropriate values from P97 we find that $\hat{q}_0 = 0.48$ for the uncorrected magnitudes and $\hat{q}_0 = 0.50$ for the corrected magnitudes. Thus we see that applying the magnitude corrections has negligible effect on the best-fit value of q_0 for the P97 data.

Figure 1 shows the value of χ^2, for $-0.5 < q_0 < 1$, both for the uncorrected (solid line) and corrected data (dashed line). We can see from this Figure that the corrected data consistently give a slightly better fit to a Friedmann model than the uncorrected data for a wide range of values of q_0. Both the uncorrected and corrected data give an acceptable fit over the range $0 < q_0 < 1$ (and in fact over a somewhat wider range for the corrected data) but both give a poor fit for large negative values of q_0.

Table 1 quantifies the goodness of fit of the SNIa data to a number of Friedmann models with different values of the cosmological parameters $\Omega_{\rm m}$, Ω_Λ, and the age of the universe, t_0 – all for a Hubble constant $H_0 = 65$ km s^{-1} Mpc^{-1}. Column (3) shows the corresponding value of q_0, calculated from Eq. (1). Column (5) gives the reduced χ^2 of the fit to all seven SNIa, while column (6) gives the reduced χ^2 obtained using the five SNIa with corrected magnitudes.

EDITOR: PLACE TABLE 1 HERE.

It is clear from Table 1 that one cannot obtain, with $H_0 \simeq 65$ km s^{-1} Mpc^{-1}, an acceptable fit to either the corrected or uncorrected data and at the same time ensure an age of the universe in excess of 14 Gyr. We discuss the situation for other values of H_0 below.

4.2. Stephani Model II

We now compare the SNIa data with the predicted magnitude-redshift relations of the Stephani models. We consider first Model II, and the relation given by Eq. (3). If we compare Eqs. (3) and (14) we see that in the limit as $z \to 0$ these equations are identical if and only if

$$\tau_0 = \frac{2}{3} H_0^{-1}. \tag{20}$$

In other words for nearby SNIa, Stephani Model II predicts the same linear redshift-magnitude relation as do Friedmann models, and with an age of the universe equal to two thirds times the inverse of the Friedmann Hubble constant. This is precisely the age of a Friedmann universe which is flat with a zero cosmological constant. In particular, if $H_0 \sim 65$ km s^{-1} Mpc^{-1} then *independent of the value of the parameter, α,* the age of the universe τ_0 in Model II is approximately 10 Gyr, which certainly appears to be too low to be consistent with astrophysical age determinations. Hence it would seem that Model II is not particularly useful in resolving the current age conflict since the age is inextricably linked to the value of the Friedmann Hubble constant: as soon as the latter is specified then so too is the age of Model II.

The link between the magnitude-redshift relation for Model II and the Friedmann case is, nonetheless, interesting for the following reason. Note that Eq. (3) may be rewritten as

$$\begin{aligned} m_{\rm B} &= M_{\rm B} + 25 - 5\log_{10}\frac{2}{3\tau_0} + 5\log_{10} cz \\ &+ 1.086(1-q_0)z, \end{aligned} \tag{21}$$

where

$$q_0 = \left(\frac{1}{2} - \frac{9}{8}c^2\alpha\tau_0^{\frac{4}{3}}\right), \tag{22}$$

which means that for any given age of the universe, τ_0, we can choose the parameter, α, so that the magnitude-redshift relation for Model II is identical in form to Eq. (14), with $\tau_0 = \frac{2}{3}H_0^{-1}$. The crucial difference is that, whereas in the Friedmann case with $\Lambda = 0$, Eq. (20) implies that $q_0 = 0.5$, in the Stephani case we still retain the freedom to specify a relation which is equivalent to *any* value of q_0 by suitable choice of α.

In particular, by choosing $\alpha < 0$ one can obtain a magnitude-redshift relation which corresponds to a Friedmann model with $q_0 > 0.5$. This is in full analogy to Friedmann models if the relation for the curvature of the Stephani models is taken into account (Dąbrowski 1995, Eq. 2.14). It shows that the time dependent curvature index (Eq. 10) for $\alpha < 0$ is positive (if cosmic time, $\tau > 0$), while for $\alpha > 0$ it is negative. The pressure (Eq. 12) is positive or negative for α being positive or negative respectively at the center of symmetry $r = 0$ and it diverges to either plus or minus infinity (depending on the values of other parameters) at the antipodal center of symmetry $r \to \infty$ and the particles are either accelerated away or towards $r = 0$. Bearing in mind the effect of curvature (Eq. 10) of the models one can roughly say that the inclusion of non-uniform pressure mimics a flat Friedman model $q_0 = 1/2$ to become curved – positively curved for $\alpha < 0$, and negatively curved for $\alpha > 0$.

While in the Friedmann case $q_0 > 0.5$ would imply an age of the universe $\tau_0 < \frac{2}{3}H_0^{-1}$, in the case of Model II we still have $\tau_0 = \frac{2}{3}H_0^{-1}$. Model II would, therefore, be of considerable interest if SNIa (or other) observations were to suggest that $q_0 > 0.5$, which certainly cannot be ruled out on the basis of the P97 data alone. As an illustrative (if somewhat extreme) example, consider the case where $\Omega_{\rm m} = 2$ and $\Omega_\Lambda = 0$, so that $q_0 = 1$ for the Friedmann model. As can be seen from Eq. (22)

the Friedmann and Model II magnitude-redshift relations are identical when

$$\alpha = -\frac{4}{9} c^{-2} \tau_0^{-\frac{4}{3}}. \tag{23}$$

Whereas the age of the Friedmann model with $q_0 = 1$ would be reduced by about 15% compared with the Einstein de Sitter age (i.e. $\tau_0 \simeq 0.57 H_0^{-1}$), for the Stephani Model II we still have $\tau_0 = \frac{2}{3} H_0^{-1}$. Although the scenario of $q_0 > 0.5$ appears highly unlikely, in view of a variety of other observations of large scale structure and CMBR anisotropies, this serves as an interesting example of how the Stephani models can be compatible with high redshift observations over a larger region of parameter space than Friedmann models.

4.3. Stephani Model I

One of the reasons why Model II is not particularly useful as an extension of the Friedmann case is that the effect of the non-uniform pressure (manifest via the parameter α) only becomes apparent at high redshift. The situation with Stephani Model I is different, however. We can see from Eq. (2) that the effect on the magnitude-redshift relation of the non-uniform pressure parameter a is immediate. In particular, therefore, even at low z Model I does not in general reduce trivially to a specific Friedmann case.

Note that after setting the parameter b from Model I equal to unity (provided $d = 0$) we can rewrite Eq. (2) to depend only upon the non-uniform pressure parameter a. Thus

$$\begin{aligned} m_{\rm B} &= M_{\rm B} + 25 + 5 \log_{10} \left[cz \left(\frac{a\tau_0^2 + \tau_0}{2a\tau_0 + 1} \right) \right] \\ &+ 1.086 \left[1 + 4 \frac{(a\tau_0^2 + \tau_0)}{(2a\tau_0 + 1)^2} \right] z. \end{aligned} \tag{24}$$

As a means of estimating what range of values of a and τ_0 will give an acceptable fit to the P97 data it is useful to note further that we may recast Eq. (24) in the form

$$\begin{aligned} m_{\rm B} &= M_{\rm B} + 25 + 5\log_{10} cz - 5\log_{10}\tilde{H}_0 \\ &+ 1.086(1-\tilde{q}_0)z, \end{aligned} \tag{25}$$

where

$$\tilde{H}_0 = \frac{2a\tau_0 + 1}{a\tau_0^2 + \tau_0} \tag{26}$$

and

$$\tilde{q}_0 = -4a\frac{a\tau_0^2 + \tau_0}{(2a\tau_0 + 1)^2}. \tag{27}$$

Eq. (24) now takes the same functional form as Eq. (14), as was similarly pointed out in Dąbrowski (1995), with $\tilde{H}_0$ and $\tilde{q}_0$ replacing H_0 and q_0. We can think of $\tilde{H}_0$ (which is one third of the expansion scalar Θ of the model) and $\tilde{q}_0$ as a generalised Hubble parameter and deceleration parameter which are related to the age of the universe in a different way from the Friedmann case. The key question of interest here is therefore whether one can construct generalised parameters, $\tilde{H}_0$ and $\tilde{q}_0$, which are in good agreement with the P97 data but which correspond to a value of τ_0 which exceeds that Friedmann age with $H_0 = \tilde{H}_0$ and $q_0 = \tilde{q}_0$. The fact that we can write the Model I redshift-magnitude relation in the form of Eq. (25) confirms, however, that our choices of τ_0 and a are certainly *not* arbitrary. Combinations of τ_0 and a which give a large negative value of $\tilde{q}_0$, for example, would clearly be incompatible with the SNIa Hubble diagram – just as was the case for Friedmann models with $q_0 << 0$.

In order to estimate the parameters τ_0 and a we construct the reduced

chi-squared statistic:-

$$\chi^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left[\frac{m_{\rm B}^{\rm obs}(i) - m_{\rm B}^{\rm pred}(i;\tau_0,a)}{\sigma(i)}\right]^2, \tag{28}$$

where n is the number of SNIa and $m_{\rm B}^{\rm pred}(i;\tau_0,a)$ is obtained for the ith SNIa from Eqs. (25), (26) and (27). We determine $M_{\rm B}$ from Hamuy et al. (1996), adopting their best-fit value of $H_0 = 63.1$ km s^{-1} Mpc^{-1} determined from four local calibrating SNIa. Thus,

$$M_{\rm B} \equiv \mathcal{M}_{\rm B} + 5\log H_0 - 25 = -19.17 \pm 0.03 \tag{29}$$

and

$$M_{\rm B,corr} \equiv \mathcal{M}_{\rm B,corr} + 5\log H_0 - 25 = -19.32 \pm 0.05. \tag{30}$$

The dependence of Eq. (28) on τ_0 and a is non-linear, making a plot of the surface $z = \chi^2(\tau_0, a)$ difficult to interpret. We therefore consider slices through this surface. Moreover for plots of χ^2 at constant τ_0 it is useful to plot χ^2 as a function of a^{-1}. Figure 2 shows χ^2 as a function of a^{-1} for $\tau_0 = 13$ Gyr, using the uncorrected P97 data. The behaviour of χ^2 is seen to be rapidly varying for values of a^{-1} around zero, but is essentially flat for all $|a^{-1}| \gtrsim 0.3$. Thus, provided $|a| \lesssim 3$ km^2 s^{-2} Mpc^{-1}, we see that the goodness of fit of Model I to the data is essentially independent of the value of a, and depends only on τ_0. A very similar curve is obtained for the corrected magnitudes.

We can understand the rapidly varying behaviour of χ^2 for small values of $|a^{-1}|$ by considering the behaviour of $\tilde{H}_0$ and $\tilde{q}_0$ in Eqs. (26) and (27). We see that when $a^{-1} = -2\tau_0$ we have $\tilde{H}_0 = 0$ and $|\tilde{q}_0| \to \infty$ [3], so that $\chi^2 \to \infty$. It therefore follows that for $\tau_0 = 13$ Gyr $= 13.26 \times 10^{-3}$ s Mpc km^{-1} a singular value of χ^2 occurs when

[3]This is a special situation, and moreover one which contradicts the observations since it would mean that the present day is exactly the turning point of the cosmic

$a^{-1} \simeq -0.026$, and χ^2 varies very rapidly close to this value. However, the range of very small a^{-1} is not of interest to us since it deviates too far from Friedmann models.

Figures 3a and 3b show plots of χ^2 as a function of a^{-1} but now with $\tau_0 = 15$ Gyr, for the corrected and uncorrected magnitudes respectively. A narrower range of values of a^{-1} is shown, in order to better illustrate the behaviour of χ^2 for small $|a^{-1}|$. We find that χ^2 again tends to infinity when $a^{-1} = -2\tau_0$, and is again essentially flat for all $|a^{-1}| \gtrsim 0.3$. Note also that the asymptotic value of χ^2 is a little smaller than that for $\tau_0 = 13$ Gyr, and moreover that there exists a narrow range of values of a for which χ^2 dips appreciably below its asymptotic value.

Figures 4a and 4b, on the other hand, show plots of χ^2 as a function of τ_0 for $a = -1.0$, for the uncorrected and corrected magnitudes respectively. This value of a is chosen to be representative of the asymptotic behaviour of χ^2; essentially the same plots would be obtained for all $|a| \lesssim 3$ km^2 s^{-2} Mpc^{-1}. We can see that Model I gives a good fit to the data for τ_0 in the range 13 to 15 Gyr.

Figures 5a and 5b show the values of $\tilde{H}_0$ and $\tilde{q}_0$ respectively as a function of τ_0, again for the representative value of $a = -1.0$. Also shown for comparison are the best-fit values of $H_0 = 63.1$ km s^{-1} Mpc^{-1} and $q_0 = 0.5$, obtained from Hamuy et al. (1996) and section 4.1 respectively. We see from Figure 5b that $\tilde{q}_0$ is almost independent of τ_0 over the range shown, increasing from $q_0 \simeq 0.04$ ($\tau_0 = 10$ Gyr) to $q_0 \simeq 0.08$ ($\tau_0 = 20$ Gyr). The dependence of $\tilde{H}_0$ on τ_0 is rather more pronounced,

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evolution. Following Eqs. (7) and (8) the energy density at the moment $\tau_0 = -1/2a$ is $8\pi G/c^4 \varrho = 48a^2$ and the pressure $8\pi Gp = \varrho(-1/3 + 1/6r^2)$. In the Friedman limit $a \to 0$ we have $\tau_0 \to \infty$, and $p = \varrho = 0$ which is an everlasting, empty, flat and static universe – a subcase of little or no physical interest.

however: good agreement with the Hamuy et al. value is found in the age range 14 to 16 Gyr.

The behaviour of $\tilde{H}_0$ and $\tilde{q}_0$ in Figures 5a and 5b makes sense when we consider the form of Eqs. (26) and (27) for $a\tau_0 << 1$. To first order in $a\tau_0$ these reduce to

$$\tilde{H}_0 = \frac{1}{\tau_0}\,(1 + a\tau_0) \tag{31}$$

and

$$\tilde{q}_0 = -4a\tau_0. \tag{32}$$

Thus we see that as $a\tau_0 \to 0$, $\tilde{H}_0 \to \tau_0^{-1}$ and $\tilde{q}_0 \to 0$.

The potential usefulness of Model I is now apparent. In the limit where $a\tau_0 \to 0$, the age of the universe in this model is increased by 50% compared with the Einstein de Sitter age, giving for example $\tau_0 = 15$ Gyr (compared with only 10 Gyr) for $H_0 \sim 65$. Of course when $a\tau_0 \to 0$, $\tilde{q}_0 \to 0$ also, so that the more meaningful comparison is that between Model I and a Friedmann model with $q_0 = 0$. Taking $\Omega_{\rm m} = 0.3$, $q_0 = 0$ and $H_0 = 65$ gives a Friedmann age of only 12.5 Gyr, however, so that the Model I age is still 2.5 Gyr greater than the Friedmann age. Taking larger values for the matter density results in a bigger difference between the Friedmann and Model I ages.

Figures 5a and 5b are indicative of the limiting behaviour of $\tilde{H}_0$ and $\tilde{q}_0$ for small $a\tau_0$. As we already remarked for the χ^2 plots, one obtains similar plots for all $|a| \lesssim 3$ km^2 s^{-2} Mpc^{-1}, with $\tilde{H}_0$ strongly varying as a function of τ_0 but $\tilde{q}_0$ much more weakly dependent on τ_0. The smaller the value of $|a|$ the closer $\tilde{q}_0$ lies to zero at given τ_0 – as is obvious from Eq. (32). The *sign* of a does have some bearing on the goodness of fit of the model to the P97 data, however. Although it can be seen from Figure 1 and Table 1 that the current data give an acceptable fit for $q_0 = 0$, the fit rapidly deteriorates for negative values of q_0. Thus, if $a > 0$ (i.e. a high pressure

region at $r = 0$ away from which particles are accelerated) then $\tilde{q}_0 \to 0$ from below, and a value of $a = 3$ km^2 s^{-2} Mpc^{-1} would imply $\tilde{q}_0 \simeq -0.2$ for $\tau_0 = 15$ Gyr, which gives only a marginally acceptable fit to the P97 data. If $a < 0$, on the other hand (i.e. a low pressure region at $r = 0$ towards which particles are accelerated) then one can obtain much better fits: e.g. $a = -3$ km^2 s^{-2} Mpc^{-1} implies $\tilde{q}_0 \simeq 0.2$ for $\tau_0 = 15$ Gyr.

We have emphasised the limiting behaviour of Model I, for small a, in order to make clear that the usefulness of the model is a fairly robust result and is not too sensitive to the exact value of a which is chosen – although it is true that negative values of a are favoured. It is particularly noteworthy that the limit $|a| \lesssim 3$, may be considered as the restriction on this parameter from the observational data and allows us not to be too far from the range where Friedmann models are valid. In other words one can obtain a good fit to the P97 data, with a significantly larger age, but without requiring that Model I departs too much from a Friedmann model.

According to Eq. (9), $|a| \lesssim 3$ km^2 s^{-2} Mpc-1 translates to a limit on the value of acceleration scalar of $|\dot{u}| \lesssim 0.66 \times 10^{-10}\, r$ Mpc^{-1}, where r is the radial coordinate in the model. Notice that, although the pressure is different from zero at the center $r = 0$ the effect of acceleration is not detectable at the center since the acceleration scalar (and of course vector) vanishes there.

If we allow slightly larger negative values for the non-uniform pressure parameter, a, then by careful choice of a and τ_0 we can obtain fits which give significant positive values of $\tilde{q}_0$, while retaining a significant difference between the Stephani and Friedmann ages. Although these fits require slightly more 'fine tuning' they are clearly in much closer agreement with the best-fit Friedmann value of $q_0 = 0.5$. Some examples of fits of this type are given in Table 2. The final two

columns of Table 2 show the age, $\tau_{\rm F}$, of the universe in a Friedmann model with $\Lambda = 0$ and with $\Omega_{\rm m} + \Omega_\Lambda = 1$ respectively.

EDITOR: PLACE TABLE 2 HERE.

Some general trends are evident from Table 2. Note that in all cases we see that as τ_0 increases and $|a|$ becomes smaller, the values of $\tilde{H}_0$ and $\tilde{q}_0$ are both reduced and the goodness of fit to the corrected and uncorrected data gradually deteriorates. For $\tau_0 \geq 16$ Gyr, the goodness of fit quickly becomes unacceptably large: although by suitable choice of a one can ensure that $\tilde{q}_0$ remains positive, the value of $\tilde{H}_0$ also reduces and overall the fit deteriorates. Further decrease in $|a|$, for $\tau_0 \geq 16$ Gyr, increases the value of $\tilde{H}_0$, but pushes $\tilde{q}_0$ closer to zero, so that the goodness of fit remains poor. This behaviour can also be easily seen from Eqs. (31) and (32).

It would seem, therefore, that an age of $\tau_0 = 15 - 16$ Gyr represents the upper age limit from Model I with the P97 data – at least if one adopts the SNIa calibration of H_0. Moreover, if subsequent analysis of larger samples of SNIa serve to tighten the limits on a positive value of $\tilde{q}_0$, then this limiting age could perhaps be reduced to $\tau_0 \sim 14$ Gyr. The important point to note, however, is that in this case the age limits on Friedmann models would *also* be reduced. As can be seen from Table 2, a value of $\tilde{q}_0 \sim 0.5$ can be well fitted by Model I with $\tau_0 = 14$ Gyr, which still represents an increase in the age of the universe of more than 3 Gyr compared with the $q_0 = 0.5$ Friedmann model with either zero cosmological constant or critical density.

5. Conclusions

In this paper we have considered the two exact non-uniform pressure spherically symmetric Stephani universes, and have compared the redshift-magnitude relations derived for these models in Dąbrowski (1995) with the recent SNIa observations of P97. We have investigated the extent to which, by suitable choice of the Stephani model parameters, we may obtain good fits of the P97 data to the predicted redshift-magnitude relations but for universes which are older than their Friedmann counterparts.

We emphasize that we have considered only the case of centrally placed observers, which results in having zero pressure gradient in our location. Although this is clearly a special case, it is mathematically the simplest possibility and merits consideration first. It can be extended relatively simply to the case of a non-centrally placed observer using the formulae given in Section 5 of Dąbrowski (1995). However, such a generalisation introduces additional model parameters which make apparent magnitude a function of both redshift and direction the sky. In principle we could have estimated these parameters in this paper, but the small size of the P97 sample would make such a parameter fit statistically meaningless. Indeed, even for the case of a centrally placed observer, ideally one should consider a much larger supernovae sample. We will consider the more general case in future, when the number of observed supernovae has significantly increased.

We have found that the age of the universe in Stephani Model II is, in fact, independent of the non-uniform pressure parameter, α, and is equal to the age of an Einstein de Sitter Friedmann model, i.e. $\tau_0 = \frac{2}{3}H_0^{-1}$. This model would be of considerable interest if the total density of the universe were greater than the critical density, since the age of the corresponding Friedmann model would then be *less* than

the Einstein de Sitter age. Since there exists no compelling observational evidence to suggest that the universe is closed, however, Model II is of limited use as it would in general predict an age which was smaller than its Friedmann counterpart with the same value of the Hubble constant.

We have shown that Stephani Model I would be of considerably greater interest, however. We have found that the redshift-magnitude relation predicted for Model I can be expressed in terms of two parameters: the age, τ_0, of the universe and the non-uniform pressure parameter a. One can write the redshift-magnitude relation in exactly the same form as in the Friedmann case, introducing an effective Hubble parameter, $\tilde{H}_0$, and deceleration parameter, $\tilde{q}_0$, which are non-linear functions of a and τ_0. We have shown that for a wide range of different values of a we can obtain good fits to the P97 data for a universe of age up to 15 Gyr, which is typically two or three Gyr greater than the corresponding Friedmann model. These fits are quite robust, requiring only that $|a| \lesssim 3$ km^2 s^{-2} Mpc^{-1}, which gives the value of the acceleration scalar $\dot{u}$ of the order $|\dot{u}| \lesssim 0.66 \times 10^{-10}\, r$ Mpc $^{-1}$, where r is the radial coordinate of the model. Then, although the pressure is not zero at the center of symmetry $r = 0$, the effect of acceleration is non-detectable at the center since the acceleration scalar vanishes there. However, this effect is easily extracted from the redshift-magnitude relation since neighbouring galaxies are not situated at the center and they necessarily experience acceleration.

The above robust fits are for the limiting case where the product $a\tau_0$ is small, and imply an effective deceleration parameter, $\tilde{q}_0$, close to zero – a value which is certainly not as yet ruled out by the P97 data, although the fit to a value of $\tilde{q}_0 = 0.5$ is currently somewhat better. By some fine-tuning of the Model I parameters, one can obtain good fits with $\tilde{q}_0 \sim 0.5$ and $\tau_0 \sim 14$ Gyr. While an age of only 14 Gyr may still be in conflict with astrophysical age determinations, the conflict is

considerably worse for Friedmann models: the age of an $H_0 \sim 65$, $q_0 \sim 0.5$ critical density Friedmann universe is only ~ 10 Gyr, and for closed models with $q_0 \sim 0.5$ the age is even smaller.

Thus, we find that Model I can give an age of the universe which is consistenly and robustly between two and three Gyr older than the oldest acceptable open or flat Friedmann models.

Since the preliminary results of P97 were first presented, there have been several important developments in the measurement of fundamental cosmological parameters. The recalibration of the RR Lyrae distance scale has revised age estimates of the oldest globular clusters to $t_0 = 11.7 \pm 1.4$ Gyr (c.f. Chaboyer et al. 1997). This undoubtedly lessens the conflict with the standard ($\Omega_0 = 1$, $\Lambda = 0$) cosmological model – particularly if one argues for a value of $H_0 \sim 55$ (c.f. Tammann 1996). If one requires a 'gestation period' of around 1 Gyr between the Big Bang and the formation of the first globular clusters, however, then agreement with the standard model is still only marginal – even for $H_0 = 55$ – and open Friedmann models would appear to be favoured. Since we have argued in this paper that the P97 data does not yet exclude models with $q_0 \sim 0$, it is only fair to point out that open Friedmann models with $\Lambda \neq 0$, $q_0 \sim 0$ and $H_0 \sim 55$ offer a comfortable resolution of the age problem, in the light of the Chaboyer et al. results. Adopting, for example, $\Omega_{\rm m} = 0.5$, $\Omega_\Lambda = 0.25$ and $H_0 = 55$ gives a Friedmann age of $\tau_{\rm F} = 14.0$ Gyr.

It is important to recognise, however, that this agreement rests crucially upon the value of H_0. If, instead, one adopts the most recent estimate of H_0 from the HST distance scale Key Project: $H_0 = 73 \pm 6 \pm 8$ km s^{-1} Mpc^{-1} (Freedman 1996), the above Friedmann age reduces to only 10.6 Gyr, and for the standard model

(with $q_0 = 0.5$) is only 8.9 Gyr. Moreover, the impact on H_0 of the HIPPARCOS recalibration of the LMC distance modulus has recently been shown by Madore and Freedman (1997) to be less significant than had previously been reported (c.f. Feast & Catchpole 1997). Thus it would seem that rumours of the end of the age problem are perhaps somewhat premature. If, indeed, the value of H_0 lies close to that obtained by the HST Key Project then we note that the Stephani models considered here can still give an age of up to approximately 12.5 Gyr, with $q_0 \sim 0.5$, and 13.4 Gyr, with $q_0 \sim 0$. While the data clearly do not yet present a case for abandoning Friedmann models, equally they do not rule out the possible need to do so in the future – when bounds on H_0 and q_0 are tightened – and the Stephani models investigated here could indeed prove to be very important.

In this paper we have considered only a particular class of inhomogeneous models, in order to illustrate their potential usefulness in addressing the apparent conflict between the observed values of the Friedmann model parameters. In future work we will extend our treatment to a wider class of non-Friedmann models and test their compatibility with the Hubble diagram of high-redshift objects and other cosmological observations. Such a comparison will be greatly enhanced by having larger samples of distant SNIa – a development which the modern observing strategies adopted by P97 and other groups will shortly provide.

As for the further progress in our comparison of the Stephani models with astronomical data one should of course investigate such standard tests as galaxy number counts, the angular size-redshift relation or microwave background anisotropies as far as to adopt the second order terms in redshift z^2. This can indeed be done after some relatively tedious calculations, again using the powerful power series methods originally given by Kristian & Sachs (1966, see also Ellis 1971 for more detailed discussion). Many of these issues have been studied previously

for Friedmann models (e.g. Dąbrowski & Stelmach 1986, 1987; Stelmach, Byrka & Dąbrowski 1990) and will be the subject of future work.

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Fig. 1.— The value of χ^2, for $-0.5 < q_0 < 1$, both for the uncorrected (solid line) and corrected data (dashed line).

Fig. 2.— Plot of the variation of χ^2 with a^{-1}, for a fixed value of $\tau_0 = 12$ Gyr, obtained from a comparison of the apparent magnitudes predicted by Stephani Model I with the uncorrected magnitudes of P97.

Fig. 3.— Plot of the variation of χ^2 with a^{-1}, for a fixed value of $\tau_0 = 15$ Gyr, obtained from a comparison of the apparent magnitudes predicted by Stephani Model I with the magnitudes of P97. Figure 3a is for the uncorrected magnitudes and Figure 3b is for the corrected magnitudes.

Fig. 4.— Plot of the variation of χ^2 with τ_0, for a fixed value of $a = -1.0$, obtained from a comparison of the apparent magnitudes predicted by Stephani Model I with the magnitudes of P97. Figure 4a is for the uncorrected magnitudes and Figure 4b is for the corrected magnitudes.

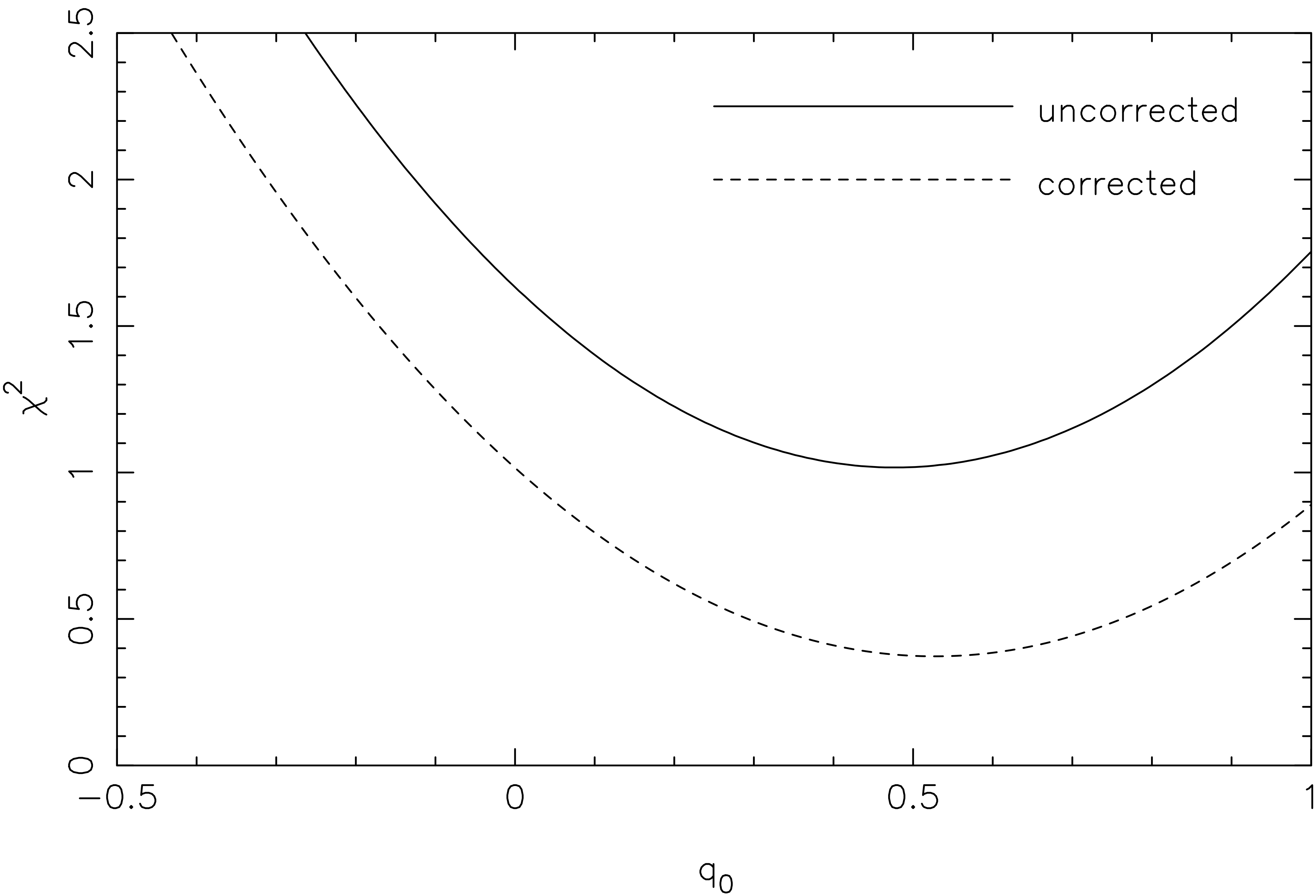
Fig. 5.— Plot of the effective Friedmann parameters, $\tilde{H}_0$ and $\tilde{q}_0$, as a function of τ_0, for a fixed value of $a = -1.0$. Dotted lines indicate the best-fit Friedmann values for H_0 (Figure 5a) from Hamuy et al. (1996) and q_0 (Figure 5b) from section 4.1.

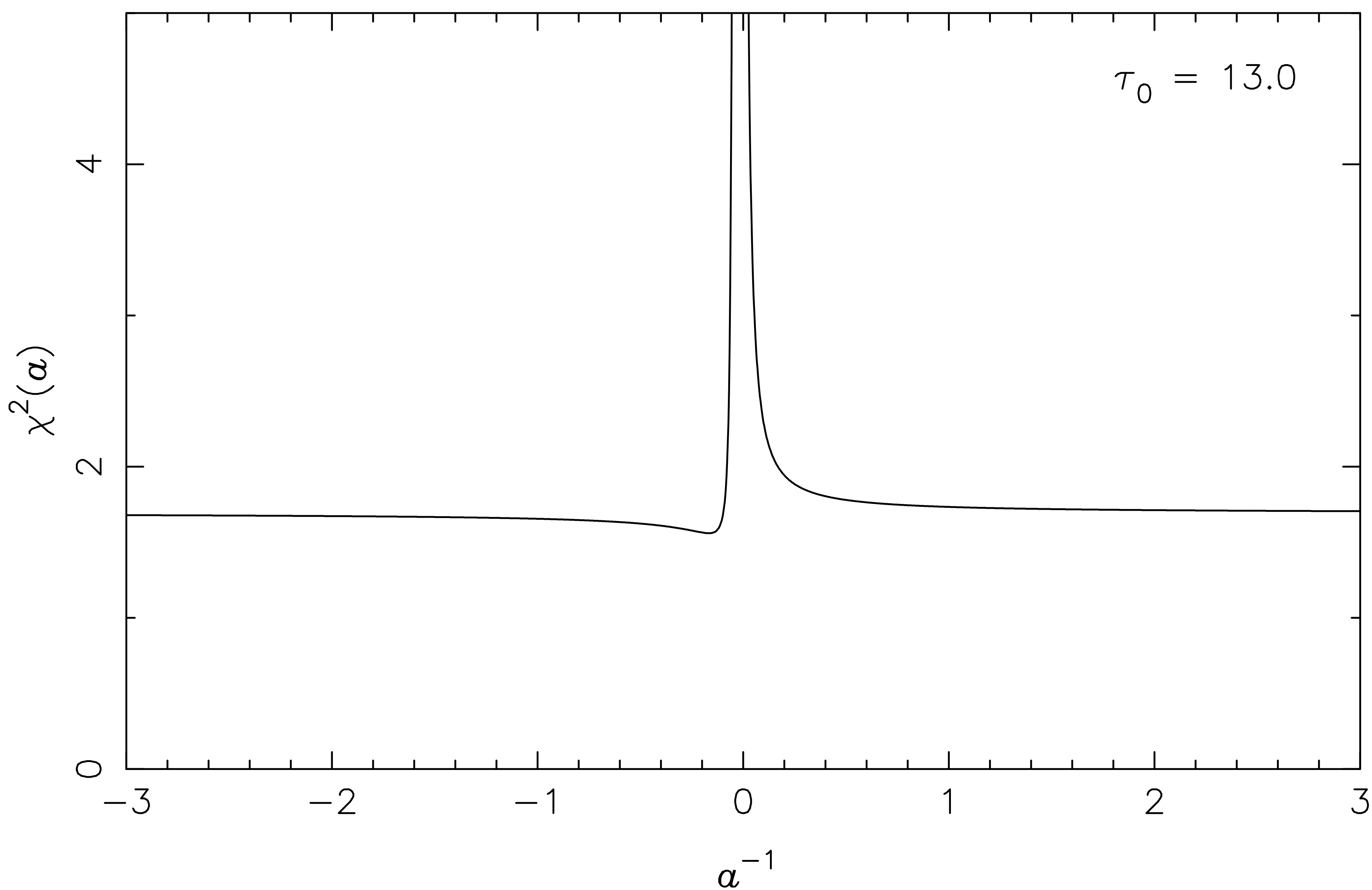
Table 1. Goodness of fit, expressed as a reduced χ^2, of the SNIa data to Friedmann models with different combinations of $\Omega_{\rm m}$ and Ω_Λ. Column (3) gives the corresponding value of q_0 and column (4) the age, t_0, of the universe assuming $H_0 = 65$ km s^{-1} Mpc^{-1}. Columns (5) and (6) respectively give the reduced χ^2 of the fit to all seven SNIa and to the five SNIa with corrected magnitudes.

$\Omega_{\rm m}$	Ω_Λ	q_0	t_0 Gyr	χ^2	$\chi^2_{\rm corr}$
1.0	0.0	0.5	10.0	0.87	0.37
0.5	0.0	0.25	11.3	0.99	0.55
0.3	0.0	0.15	12.2	1.12	0.70
0.2	0.0	0.1	12.7	1.20	0.79
0.3	0.15	0.0	12.9	1.40	1.02
0.2	0.1	0.0	13.4	1.40	1.02
0.3	0.25	-0.1	12.8	1.68	1.32
0.5	0.5	-0.25	12.5	2.09	1.77
0.3	0.7	-0.55	14.5	3.31	3.06
0.2	0.8	-0.7	16.2	4.07	3.86

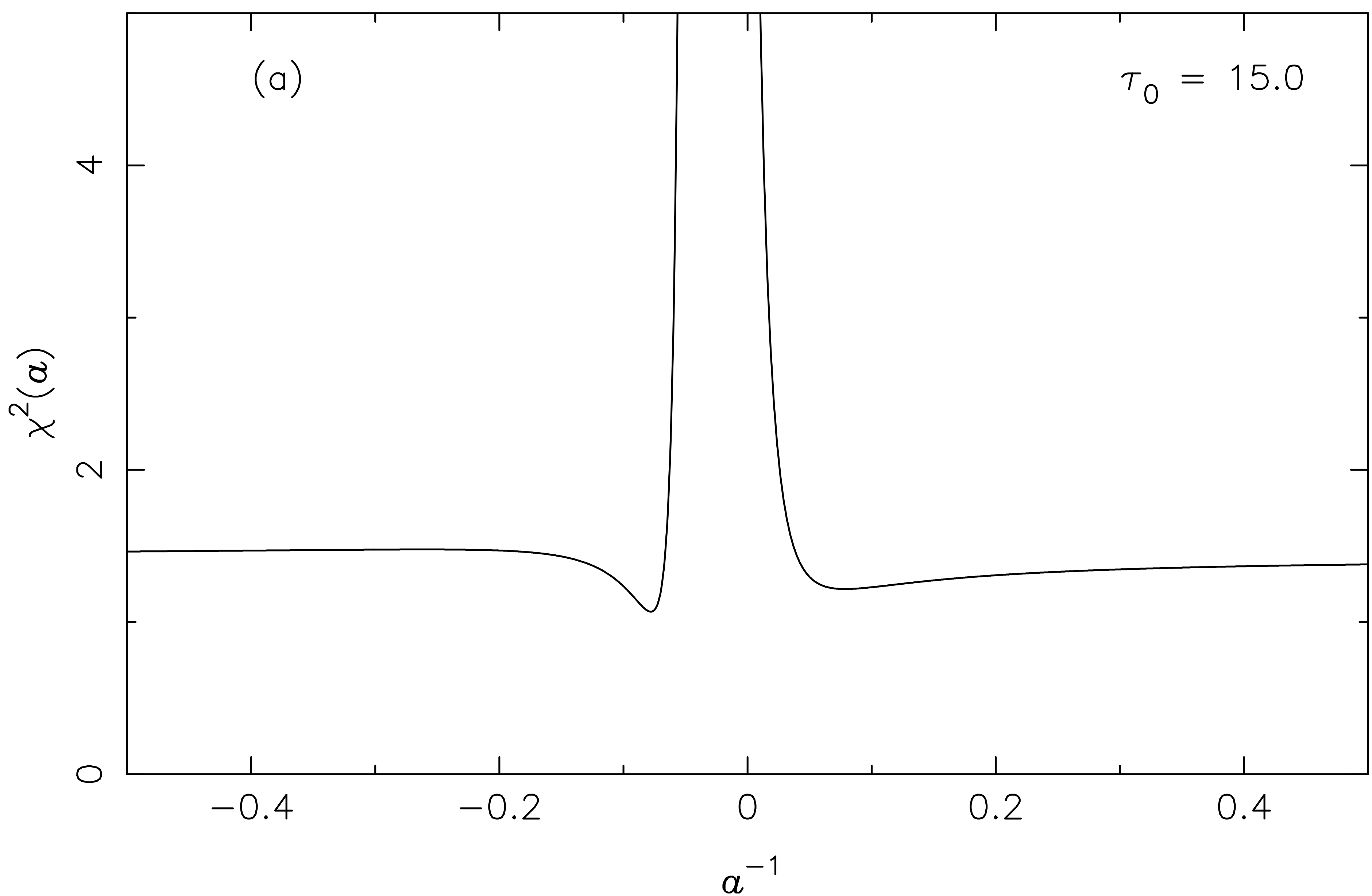
Table 2. Fits of Stephani Model I universes with significantly positive values of $\tilde{q}_0$ to the P97 data. The final two columns show the age, τ_F, of the universe in a Friedmann model with the same values of H_0 and q_0, with $\Lambda = 0$ and with $\Omega_m + \Omega_\Lambda = 1$ respectively.

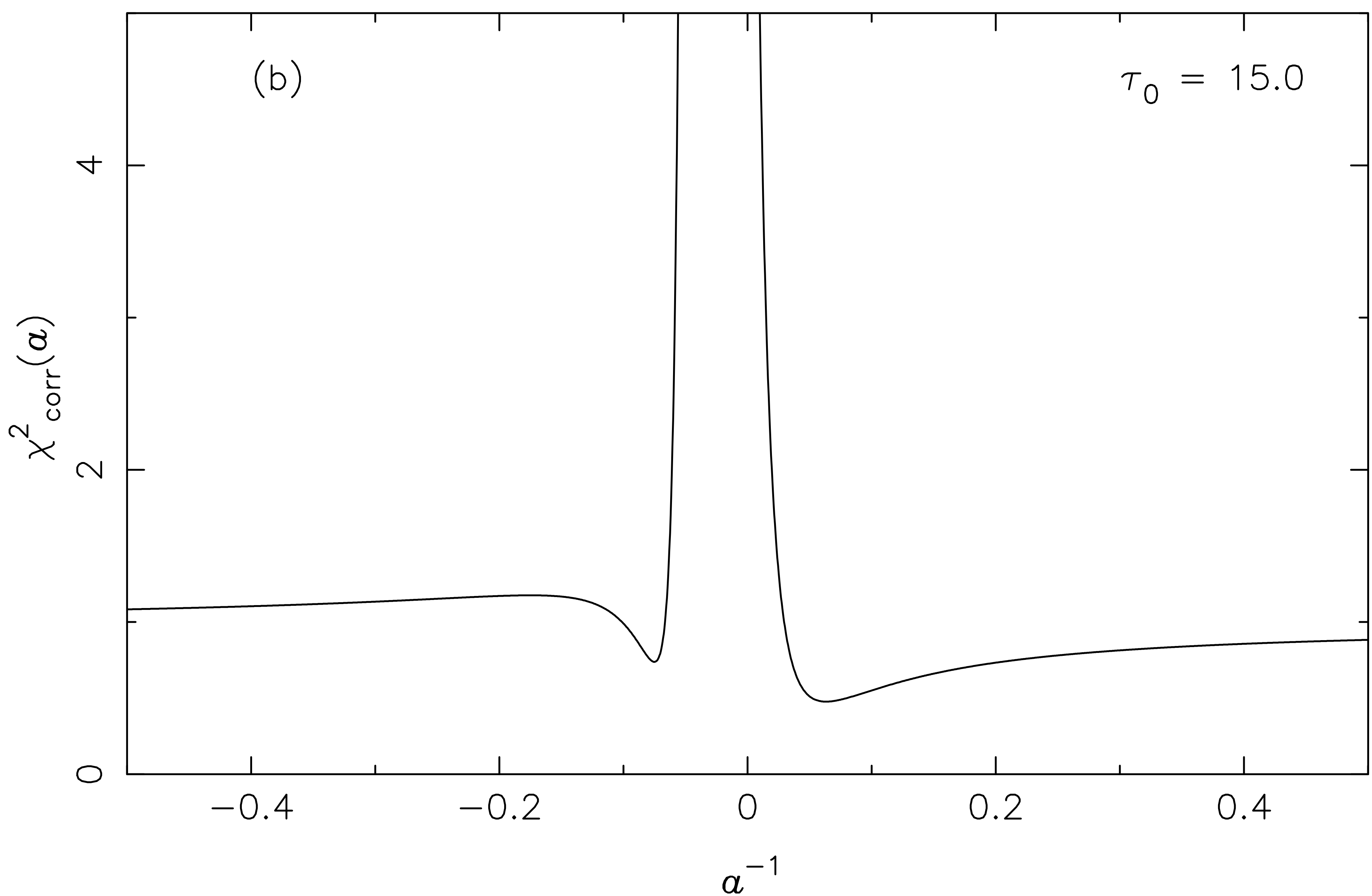
τ_0 Gyr	a km^2 s^{-2} Mpc^{-1}	$\tilde{H}_0$ km s^{-1} Mpc^{-1}	$\tilde{q}_0$	χ^2	$\chi^2_{\rm corr}$	τ_F ($\Lambda = 0$) Gyr	τ_F ($\Omega_m + \Omega_\Lambda = 1$) Gyr
13.00	-10.00	63.7	0.86	1.67	1.07	9.1	9.5
13.25	-8.33	64.4	0.67	1.40	0.79	9.5	9.8
13.50	-7.14	64.5	0.55	1.24	0.64	9.9	10.0
13.75	-6.67	63.8	0.51	1.15	0.56	10.2	10.2
14.00	-6.25	63.0	0.48	1.12	0.58	10.4	10.3
14.25	-5.55	62.6	0.42	1.14	0.65	10.8	10.6
14.50	-5.00	62.0	0.38	1.22	0.77	11.1	10.8
14.75	-4.55	61.4	0.34	1.32	0.94	11.4	11.0
15.00	-4.17	61.3	0.27	1.48	1.14	11.9	11.2
15.25	-3.85	60.0	0.29	1.66	1.42	12.0	11.4
15.50	-3.33	59.6	0.25	1.88	1.72	12.4	11.6
15.75	-3.12	58.8	0.23	2.14	2.07	12.7	11.8
16.00	-0.35	58.1	0.22	2.43	2.45	12.9	12.0

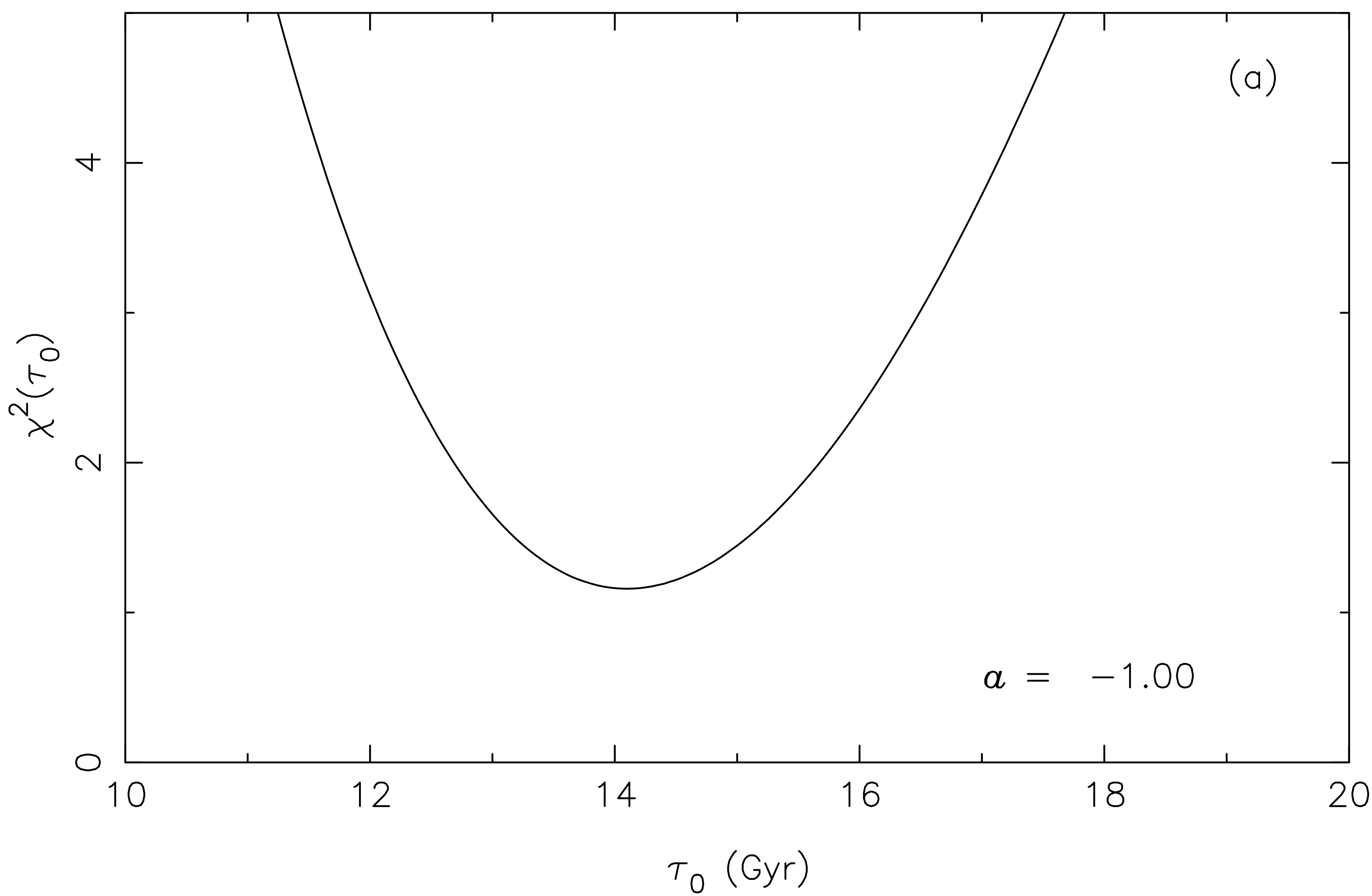

uncorrected
corrected
2.5
2
1.5
1
0.5
0
−0.5
0
0.5
1
χ^2
q_0

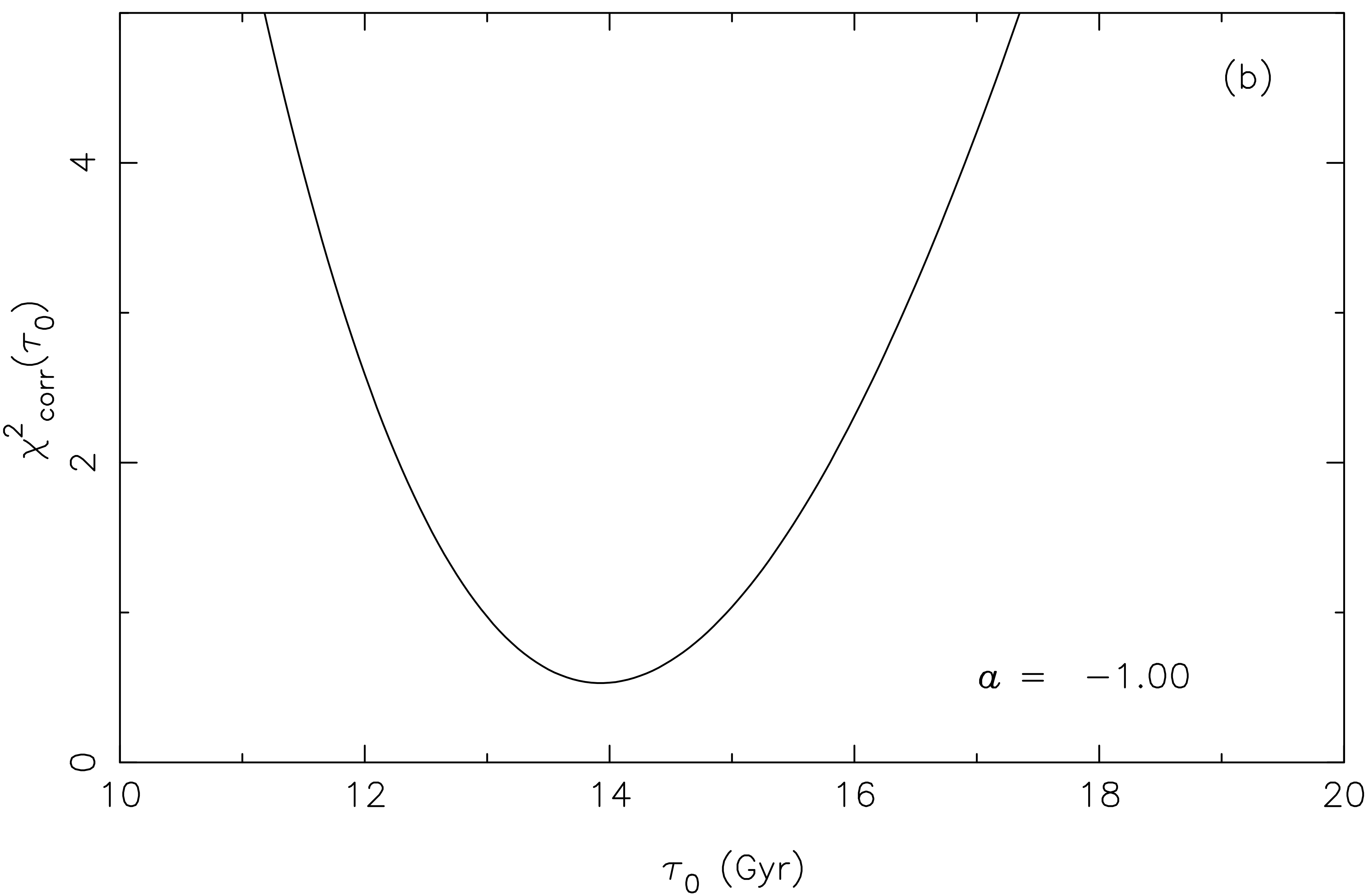


$\tau_0 = 13.0$
$\chi^2(a)$
0
2
4
−3
−2
−1
0
1
2
3
a^{-1}

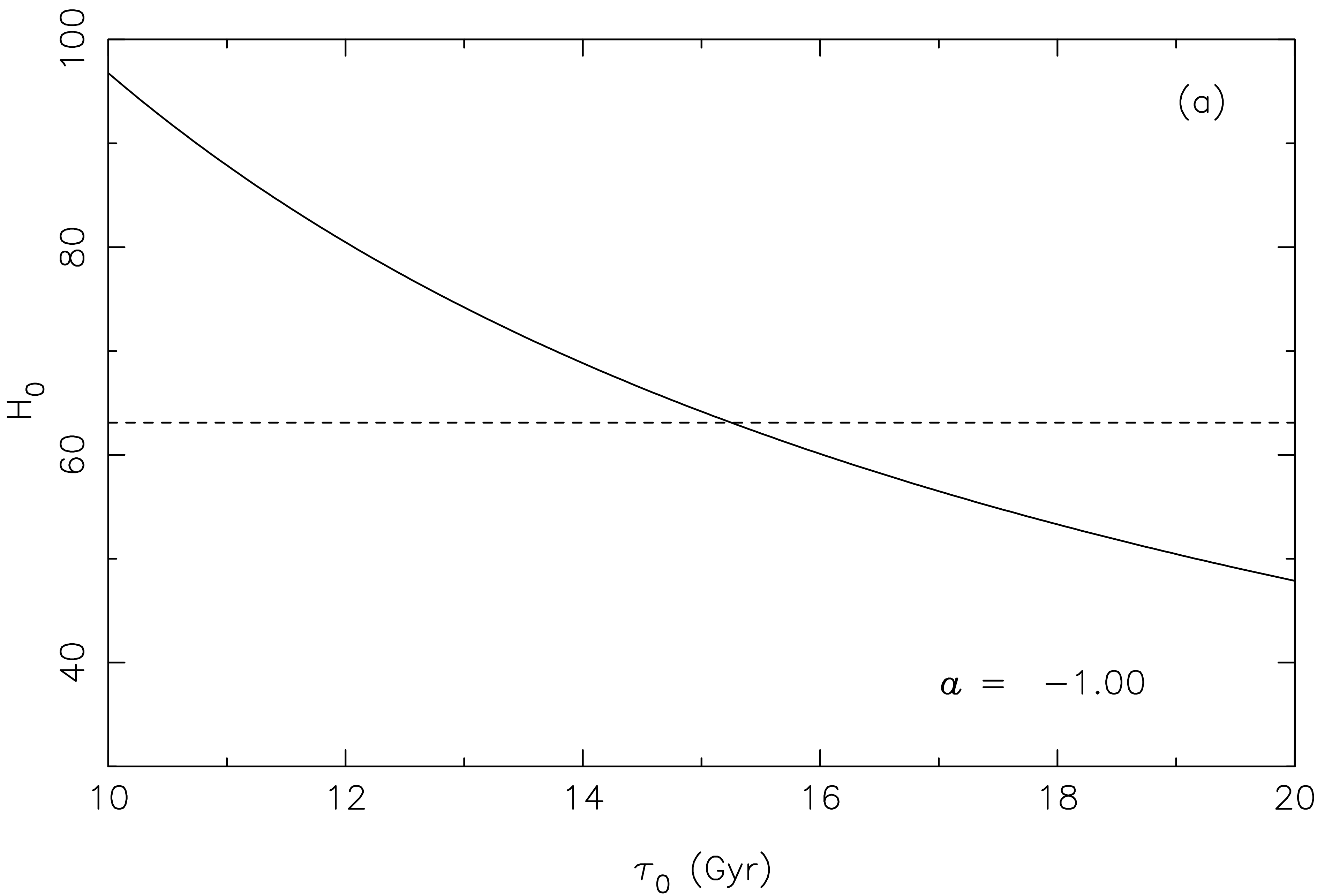

(a)
$\tau_0 = 15.0$
4
2
0
$\chi^2(a)$
−0.4
−0.2
0
0.2
0.4
a^{-1}

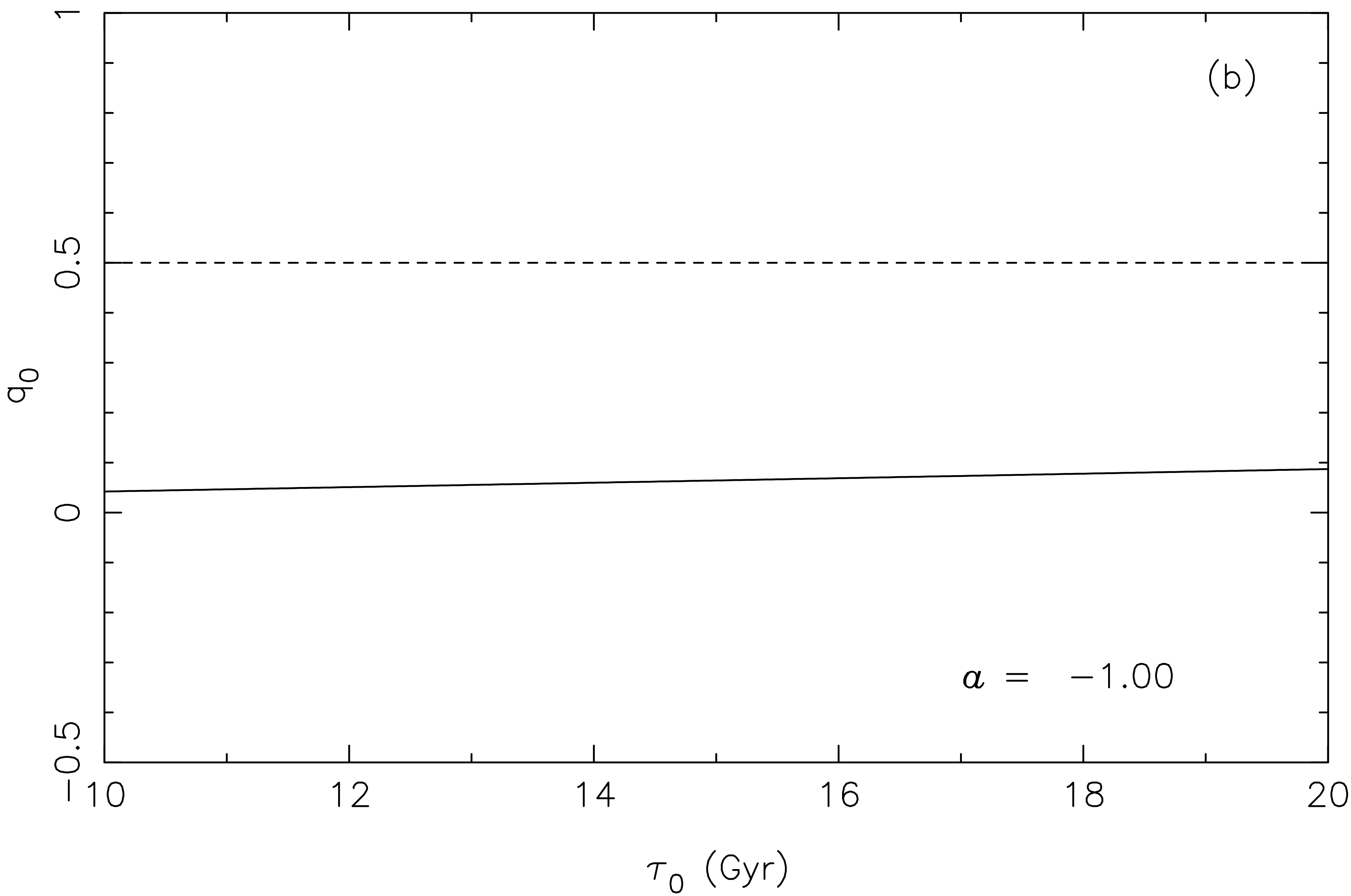

(b)
$\tau_0 = 15.0$
$\chi^2_{corr}(\alpha)$
0
2
4
−0.4
−0.2
0
0.2
0.4
α^{-1}


(a)
α = −1.00
$\chi^2(\tau_0)$
τ_0 (Gyr)
0
2
4
10
12
14
16
18
20



(b)
$a = -1.00$
$\chi^2_{corr}(\tau_0)$
τ_0 (Gyr)
0
2
4
10
12
14
16
18
20


(a)
α = −1.00
H_0
100
80
60
40
10
12
14
16
18
20
τ_0 (Gyr)



(b)
α = −1.00
q0
τ0 (Gyr)
1
0.5
0
−0.5
10
12
14
16
18
20